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Scope of the Meeting

The conference Finite Element Methods in Engineering and Science (FEMTEC 2006) is supported by the NSF, University of Texas at El Paso (UTEP), and IMACS. Its main goal is to advance the frontiers in performance and reliability of finite element methods, as well as in their application to computational engineering and science.

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Contact Information

E-mail: `femtec@math.utep.edu`
Web page: `http://www.math.utep.edu/femtec_2006/`
Mailing address:
FEMTEC 2006,
Department of Mathematical Sciences,
500 West University Avenue,
El Paso, TX 79968

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Part I

Keynote Speakers

Reliability of Computational Science. A Particular Problem

I. Babuška
University of Texas at Austin
babuska@utaustin.edu

F. Nobile
Polytecnico di Milano
nobile@milano.it

R. Tempone
Florida State University at Tallahassee
tempone@talla.edu

Abstract

Engineering decisions are often based on the solution of a mathematical problem. Mathematical problem has a structure, input and the output (the quantity of interest). It does nothing else that it transforms the available information in the desired one (the quantity of interest). The available information is related to the experimental data. They are usually insufficient. Hence in the formulation and solution of the problem the uncertainty in the available data has to be taken into consideration. The talk will address a particular frame problem which illustrate the difficulties and methodology how to deal with problems of this kind. In this problem the goal is to obtain the probability that the deflection (the quantity of interest) will not exceed 3 mm and quantitative characterization of the confidence in the computed probability. For disposition are the data for the calibration, validation and accreditation experiments. The data belong to the three groups, which differ in the number of the available experiments. It will be seen how the reliability of the prediction depends on the number of available experiments.

On Finite Element Methods for Crack Propagation and Dislocations

T. Belytschko
Department of Mechanical Engineering
Northwestern University
Evanston, Illinois 60208 U.S.
tedbelytschko@northwestern.edu

Abstract

The formulations and performance of various methods for static and dynamic crack propagation will be presented and examined. Among the methods considered are element deletion, interface cohesive crack methods, new meshfree methods and the extended finite element [1–3]. For benchmarks, the performance of these methods in problems with both stable and unstable crack paths are studied. It has been found that there are substantial differences in the crack paths predicted by the various methods and that the element deletion method does not reproduce unstable crack paths in brittle materials very well. Differences in the crack speeds are also examined. The development of a simple fast method for modeling dislocations [4] based on the extended finite element method is described. In this method, the dislocation is modeled by an interior discontinuity. Applications to problems with interfaces, which are difficult by the standard methods based on superposition, are desribed.

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4. R. Gracie, G. Ventura and T. Belytschko, A new fast finite element method for dislocations based on interior discontinuities, International Journal for Numerical Methods in Engineering, to appear 2006.

Mimetic Discretizations and What They Can Do For You

P. Bochev[1][2]
Sandia National Laboratories, P.O. Box 5800, MS 1110,
Albuquerque, NM 87185-1110
pbboche@sandia.gov

Abstract

In the past few years homological ideas have been gaining an increased attention from the numerical community. Thanks to their use, we now have a much better understanding of why some discretization methods work so well and why other methods fail spectacularly. More importantly, homological ideas can be used to develop stable and physically consistent discretizations for a large class of PDEs. One such approach is mimetic methods which replace PDEs by algebraic equations that mimic their fundamental structural properties. We provide a common framework for mimetic methods using algebraic topology to guide our analysis. The key concept in our approach is the natural inner product on co-chains. This inner product is sufficient to generate a combinatorial Hodge theory on co-chains but avoids complications attendant in the construction of robust discrete Hodge-star operators. In particular, using a reduction and a reconstruction maps between differential forms and co-chains we define mutually consistent sets of natural and derived discrete operations that preserve the invariants of the De Rham homology groups and obey a discrete Stokes theorem. By choosing a specific reconstruction operator we obtain well-known mixed FE, mimetic FD and covolume methods and explain when they are equivalent. In the second half of the talk we will present several applications of the mimetic framework. We will start with a new interpretation of a certain class of compatible least-squares methods, as discrete realizations of a Hodge * operator obtained from weakly enforced material laws. Among other things, we demonstrate that least-squares, Galerkin and mixed Galerkin methods for second order elliptic problems can all be derived from a common constrained optimization problem. Then we will derive mimetic discretizations of the eddy current equations and show how to develop efficient and robust algebraic multigrid solvers for them. We will conclude with an example that illustrates how mimetic discretizations can be used to remap divergence free fields without using advection algorithms. This talk is based on joint work with M. Gunzburger (CSIT, Florida State University), M. Shashkov and M. Hyman (Theoretical Division, Los Alamos National Laboratory).

[1] Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed-Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC-94AL85000.

[2] This work was partially funded by the Applied Mathematical Sciences program, U.S. Department of Energy, Office of Energy Research.

The State of the Art of Constructing Cubature Formulas for Multi-Variate Integrals

Ronald Cools
Dept. of Computer Science, K.U.Leuven, Celestijnenlaan 200A, B-3001 Heverlee, Belgium.
Ronald.Cools@cs.kuleuven.be

Abstract

A *cubature formula* is an approximation of a multi-variate integral by a weighted sum of function values. Several criteria are used to construct such approximations. The best known criterion is probably that of *(algebraic) degree*, indicating that the approximation is exact for polynomials up to that degree.

In the 1970-80's many cubature formulas were constructed for low dimensional standard regions using that criterion. Several techniques were developed which we classify in two classes: the ideal theoretical approach and the invariant theoretical approach, see [1]. In practice both approaches turned out to be very limited. In recent years some old methods were used again and simply because computers became more powerful, new results were obtained. Progress even for 2- and 3-dimensions and standard regions such as a cube or simplex was rather small, as can be seen in the online overview `www.cs.kuleuven.be/~nines/ecf/` described in [3] We will sketch the fundamentally different approaches, emphasizing their merits and limitations. A brief overview of what happened in this research area during the second half of the twentieth century is given in [2].

Recently some new theoretical approaches were developed for constructing cubature formulas exact for polynomials. Also quasi-Monte Carlo methods are nowadays widely used. The latter are however developed with a totally different quality criterion in mind and are developed for hypercubes only. For these reasons they are probably not much used in the areas where 'classical' cubature formulas of algebraic degree proved successful in the past. Using some transformations quasi-Monte Carlo methods can also be used for simplices and the entire space. We will point the attention of the audience to these recent trends.

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Multi-Element Polynomial Chaos: Algorithms and Applications

George Karniadakis
Division of Applied Mathematics, Brown University, 182 George Street Providence, RI 02912
gk@dam.brown.edu

Abstract

The classical polynomial chaos (Wiener-Hermite expansions) appropriate for representing mostly Gaussian stochastic processes was extended by Xiu & Karniadakis (2002) to a wide class of PDFs using orthogonal polynoamials from the Askey family. More recent extensions by Wan & Karniadakis (2006) allow optimum representation of arbitrary PDFs, hence providing an effective method for multi-element decomposition as well as facilitating more easily incorporation of experimental data.

We will present an overview of generalized polynomial chaos in conjunction with Galerkin and collocation projections. Open issues related to long-time integration, parametric discontinuities, and high-dimensional stochastic inputs wil be addressed. Examples from fluid mechanics will be given including a theoretical and numerical study of scattering of shock waves by randomly rough surfaces.

References

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Interval Finite Element Methods: An Overview and Latest Development

Rafi L. Muhanna
Department of Civil and Environmental Engineering
Georgia Institute of Technology
rafi.muhanna@gtsav.gatech.edu

Abstract

Latest scientific and engineering advances have started to recognize the need for defining new models for uncertainty treatment. Probabilistic modeling cannot handle situations with incomplete or little information on which to evaluate a probability, or when that information is nonspecific, ambiguous, or conflicting. During last two decades, a number of interval-based uncertainty models for solid and structural mechanics have been developed to treat such situations.

This talk will present an overview about the Interval Finite Element Method (IFEM) and its latest development. In the formulations of IFEM, the system parameters are introduced as bounded intervals with unknown distributions. The most challenging issue in these formulations is the control of overestimation due to the well known dependency problem in interval arithmetic.

The formulation of IFEM solution techniques can be broadly classified into two groups, namely the optimization approach and the non-optimization approach. In the optimization approach, optimizations are performed to compute the minimal and maximal system responses when the uncertain parameters are constrained to belong to intervals. The non-optimization approaches for interval finite element analysis have been developed in a number of works. The emphasis of these approaches is the use of different available interval techniques to solve the resulting system of interval equations with focus on overestimation control. Illustrative examples for these approaches are the combinatorial, convex, parametric, element-by-element, and load equivalent methods.

Certified Error Bounds for Uncertain Elliptic Equations

Arnold Neumaier
Faculty of Mathematik, University of Vienna
Nordbergstr. 15, A-1090 Vienna, Austria
Arnold.Neumaier@univie.ac.at

Abstract

The results in [1] showed that is it now feasible to get realistic worst case error bounds for large uncertain linear systems of equations. However, to apply the techniques to partial differential equations, one needs to cope not only with the uncertainty due to parameters in the equations but also with the errors introduced by the discretization. In this talk we discuss the problems that need to be overcome, and possible solutions.

Using tools from functional analysis, global optimization and convex programming, methods are presented for obtaining certificates for pointwise or normwise error bounds on the solution of linear elliptic partial differential equations in polygonal domains, or of key functionals of the solutions, given an approximate solution.

Deterministic uncertainty in the parameters specifying the partial differential equations can be taken into account, given approximate solutions for a suitable sample of parameter values. The resulting bounds cover the worst case of the uncertainty.

Partial numerical results will be given for the Poisson equation.

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Part II

Contributed Lectures

Advances in *A Posteriori* Error Estimation for Discontinuous Galerkin Methods

Slimane Adjerid
Department of Mathematics, Virginia Tech, Blacksburg, VA, 24061
adjerids@math.vt.edu

Baccouch Mahboub
Department of Mathematics, Virginia Tech, Blacksburg, VA, 24061
baccouch@math.vt.edu

Abstract

Discontinuous Galerkin methods (DGM) have gained in popularity during the last 20 years because of their ability to address problems having discontinuities, such as those that arise in hyperbolic conservation laws. The DGM use a discontinuous finite element basis which simplifies *hp* adaptivity and leads to a simple communication pattern across faces that makes them useful for parallel computation. In order for the DGM to be useful in an adaptive setting, techniques for estimating the discretization errors should be available both to guide adaptive enrichment and to provide a stopping criteria for the solution process. We will present new superconvergence results on two-dimensional meshes and show how to construct effective estimates of the finite element discretization error using superconvergence of DG solutions.

We will present several new $O(h^{p+2})$ pointwise superconvergence results for first-order hyperbolic problems on triangular meshes consisting of one-outflow-edge elements as well as on meshes having both one- and two-outflow-edge elements. We will present efficient techniques to compute asymptotically correct *a posteriori* error estimates.

Finite Element Analysis of a Convection-Diffusion Equation

Jose Avila
Department of Mathematical Sciences The University of Texas at El Paso 500 W. University Ave., El Paso, Tx., 79968.
jjavila@utep.edu

Pavel Solin
Department of Mathematical Sciences The University of Texas at El Paso 500 W. University Ave., El Paso, Tx., 79968.
solin@utep.edu

Abstract

When a solid object interacts with a flowing medium (such as, e.g., water or air), the molecules of the fluid change their velocity very rapidly within a thin film adjacent to the object's surface. This film is called *boundary layer*. Due to the extremely small width of boundary layers, their numerical approximation is challenging. In particular, due to the hyperbolic nature of the equations of fluid dynamics, numerical errors committed in the boundary layer are transported quickly into the rest of the computational domain, and they may spoil the results of the computation completely. Therefore, accurate and efficient approximation of flows in boundary layers is a topic of paramount importance in aerospace, air force, and naval research.
In this work we analyze a model linear convection-diffusion equation which exhibits a boundary layer, and study the optimality of meshes for its finite element approximation. So far, the best meshes available are the *Shishkin* and *Bakhvalov* meshes. We construct a new class of meshes which are based on the equidistribution of the piecewise-linear interpolation error in the finite elements. Numerical results show that such meshes have better approximation properties than both the Shishkin and Bakhvalov meshes.

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Approximate Dirichlet Boundary Conditions in the Generalized Finite Element Method

Ivo Babuška
University of Texas at Austin, TX 78712-0027
Babuska@ticam.utexas.edu

Victor Nistor
Penn State University, University Park, PA 16802
nistor@math.psu.edu

Nicolae Tarfulea
Purdue University Calumet, Hammond, IN 46323
tarfulea@calumet.purdue.edu

Abstract

In the past few years meshless methods for numerically solving differential equations have become increasingly attractive, especially in the engineering and physics communities. The reason behind the development of such methods is represented by the mesh generation difficulties, particularly when the domain's geometry is complicated.

One of the major problems of all meshless methods is the enforcement of Dirichlet boundary conditions. We propose a method for treating the Dirichlet boundary conditions in the framework of the Generalized Finite Element Method (GFEM), a method originated in the work of Babuška–Caloz–Osborn [1]. We use approximate Dirichlet boundary conditions and polynomial approximations of the boundary. Our sequence $\{S_\mu\}$ of GFEM-spaces is such that $S_\mu \not\subset H_0^1(\Omega)$, and so it does not conform to one of the basic FEM conditions. Let h_μ be the typical size of the elements defining S_μ and let $u \in H^{m+1}(\Omega)$ be the solution of the Dirichlet problem $-\Delta u = f$ in Ω, $u = 0$ on $\partial\Omega$, on a smooth, bounded domain Ω. Assume that $\|v_\mu\|_{L^2(\partial\Omega)} \le C h_\mu^m \|v_\mu\|_{H^1(\Omega)}$, for all $v_\mu \in S_\mu$, and $|u - u_I|_{H^1(\Omega)} \le C h_\mu^m \|u\|_{H^{m+1}(\Omega)}$, $u \in H^{m+1}(\Omega) \cap H_0^1(\Omega)$, for a suitable $u_I \in S_\mu$. Then we prove that we obtain quasi-optimal rates of convergence for the sequence $u_\mu \in S_\mu$ of GFEM approximations of u, that is, $\|u - u_\mu\|_{H^1(\Omega)} \le C h_\mu^m \|f\|_{H^{m-1}(\Omega)}$. Next, we indicate an effective technique for constructing sequences of GFEM-spaces satisfying our conditions using polynomial approximations of the boundary. Finally, we extend our results to the inhomogeneous Dirichlet boundary value problem $-\Delta u = f$ in Ω, $u = g$ on $\partial\Omega$. See [2] for more information and details.

References

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Adaptive Finite Element Methods for Cahn-Hilliard Equations

Ľubomír Baňas
Department of Mathematics, South Kensington Campus, Imperial College London, SW72AZ London, UK
l.banas@imperial.ac.uk

Robert Nürnberg
Department of Mathematics, , South Kensington Campus, Imperial College London, SW72AZ London, UK
robert.nurnberg@imperial.ac.uk

Abstract

Cahn-Hilliard equations form a base for phase-field modelling and numerical simulations of separation processes involving surface diffusion or phase separation, such as, e.g. void electromigration [1]. We discuss adaptive methods in space and time that can be used to increase effectiveness and reliability of finite element computations involving Cahn-Hilliard equations. Next, we briefly describe possible methods for the solutions of the discrete system of algebraic equations that results from the finite element discretization. Finally, we present some numerical experiments in 2D and 3D to illustrate the performance of the adaptive strategies.

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Numerical Solutions of Fuzzy Differential Equations

Barnabas Bede
Department of Mathematical Sciences
The University of Texas at El Paso
500 W. University Ave., El Paso, Tx., 79968
bbede@utep.edu

Abstract

Our information on the behavior of a real-world system is often subject of non-statistical uncertainty. In order to obtain a more realistic (not more exact!) model we have to take into account these uncertainties. Fuzzy differential equations (FDE) are a natural way to model dynamical systems under possibilistic uncertainty.

There are several approaches to the study of FDEs (see [2], [3]). The generalized differentiability concept is introduced and studied in [1]. This concept allows us to solve some shortcomings of the above mentioned methods, so in the present work, this interpretation will be used. Under appropriate conditions, the fuzzy initial value problem considered under this interpretation has locally two solutions and the successive iterations $y_{n+1}(x) = y_0 + \int_{x_0}^{x} f(t, y_n(t))dt$ and $\overline{y}_{n+1}(x) = y_0 \ominus (-1) \cdot \int_{x_0}^{x} f(t, \overline{y}_n(t))dt$, with $\overline{y}_0(x) = y_0(x) = y_0$, converge to these solutions respectively.

Numerical solution of a FDE is obtained now in a natural way, by extending the existing classical methods to the fuzzy case. The local existence of two solutions implies that we have to associate with our numerical method a local choice function between the two solutions. In the present work the following choice functions are discussed: choosing always the "old" Hukuhara differentiable solution, selecting the solution with decreasing uncertainty whenever it exists, choice function based on the principle that uncertainty increases or decreases together with the absolute value and finally, the choice function maintains the uncertainty greater than a threshold value.

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Regularity and Error Estimates for a Finite Element Method for Viscous Incompressible Stokes Flows in Nonsmooth Domains

Michal Beneš
Department of Mathematics,
Czech Technical University in Prague,
Thákurova 7, 166 29 Prague 6, Czech Republic

e-mail: benes@mat.fsv.cvut.cz

Abstract

We consider the stationary Stokes system with mixed boundary conditions in polygonal domain. Let $\Omega \subset \mathbf{R}^2$ be a bounded domain, $\partial\Omega \in \mathcal{C}^{0,1}$ and $\partial\Omega = \Gamma_1 \cup \Gamma_2$ such that Γ_1 and Γ_2 are closed, sufficiently smooth, 1-dimensional measure of $\Gamma_1 \cap \Gamma_2$ is zero and 1-dimensional measure of Γ_1 is positive. We prescribe the non-slip boundary condition on Γ_1 and the boundary condition

$$-\mathcal{P}\mathbf{n} + \frac{\partial \mathbf{u}}{\partial \mathbf{n}} = 0$$

on Γ_2. Here $\mathbf{u} = (u_1, u_2)$ is velocity, $\mathcal{P}$ represents pressure and $\mathbf{n} = (n_1, n_2)$ is an outer normal vector. We consider corner points on boundary, where the boundary conditions change their type. The weak solution to the Stokes system with mixed boundary conditions in a polygonal domain belongs to weighted Sobolev spaces. Regularity results are contained in [1] and [2]. The regularity results are important for an error analysis of numerical methods, i.e. the regularity of the weak solution has a great influence over the rates of convergence for finite element methods. We present finite element error estimates for meshes depending on regularity.

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Fully Coupled Model of Hygro-Thermal Behaviour of Concrete During Fire

Michal Beneš
Department of Mathematics,
Czech Technical University in Prague,
Thákurova 7, 166 29 Prague 6, Czech Republic
benes@mat.fsv.cvut.cz

Petr Mayer
Department of Mathematics,
Czech Technical University in Prague,
Thákurova 7, 166 29 Prague 6, Czech Republic
pmayer@mat.fsv.cvut.cz

Abstract

Heat and mass transfer in concrete structures subject to high temperatures are of great interest in safety evaluation against fire in civil engineering. The need for a new modelling capacity for concretes under such extreme conditions has been evidenced.

In this paper, we present a nonlinear mathematical model for numerical analysis of the hygro-thermal behavior of the concrete wall subject to transient heating according to the standard ISO fire curve and radiative heating (1273,15 K). This examples allows us to analyse and better understand physical and chemical phenomena taking place in concrete exposed to high temperatures.

The governing equations of the present model are the dry air conservation equation, water species (liquid - vapour) conservation equation and energy conservation equation (general nonlinear heat equation). Dry air, water vapour and their mixture are assumed to behave as perfect gases, therefore Dalton's law and the Clapeyron equation are assumed as state equations. Water vapour pressure, p_{gw} is obtained from the Kelvin equation. As the constitutive equations for fluid phases (capillary water, gas phase) the multiphase Darcy's law has been applied. The numerical algorithm connecting finite element method for the numerical solution of the energy equation and Euler method for the mass balance equations is presented. Developments of temperature, saturation and water vapour pressure are demonstrated.

An interesting phenomenon, very specific for concrete exposed to high temperatures, is the so-called thermal spalling, which can sometimes be explosive. Its physical causes are still not fully understood. One of the phenomena is considered to explain this transient: generation of internal vapor pressures, which exceed the local tensile strength of the material.

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hp-FEM for Coupled Problems – First Steps

Jakub Cerveny, Pavel Solin
The University of Texas at El Paso, 500 West University, El Paso, TX 79912
solin@utep.edu

Ivo Dolezel
Institute of Thermomechanics, Academy of Sciences of the Czech Republic,
Dolejskova 5, Praha, CZ-182 00, Czech Republic2
dolezel@fel.cvut.cz

Abstract

While numerical methods for single-field problems have reached a high degree of maturity already, computational methods for coupled problems have been far less developed. The reason is that the solution of a coupled problem is much more difficult compared to the solution of any single-field problem involved:

- various physical fields such as, e.g., the temperature, velocity, pressure, electric field, or magnetic field exhibit important qualitative differences which make a uniform computational approach very difficult or impossible,
- physical fields generally belong to different spaces of functions where different types of finite elements are needed.

In this talk we present preliminary results and main ideas of our research whose goal is to address both issues mentioned above: In order to resolve individual phenomena in various physical fields efficiently, we consider every solution component on an individual mesh where, moreover, individual adaptive strategy is employed. At the same time, every physical field is discretized using higher-order finite elements which conform to the corresponding Sobolev space. Our preliminary results are related to incompressible flow with heat transfer in 2D, where all filelds v_1, v_2, p, θ are discretized on different meshes. We demonstrate the large computational potential of such approach and mention some exciting related open problems.

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An Enriched Finite Element Method for Modeling Phase Growth

Jack Chessa
Department of Mechanical Engineering, The University of Texas at El Paso
El Paso, Texas 79968 U.S.
jfchessa@utep.edu

Manny Gonzalez
Department of Mechanical Engineering, Northwestern University
Evanston, Illinois 60208 U.S.
mgonzales@northwestern.edu

Abstract

An enriched finite element method for modeling the phase growth in oxidizing materials will be presented. Phase growth for typical oxidizing materials is governed by the oxygen concentration gradient in the phase and also the jump in the concentration at the phase-matrix interface. It is this jump which controls the surface oxidation reaction rate.

The extended finite element method (X-FEM) is employed to capture the discontinuity in the oxygen concentration [1,2]. This approach is similar to the solidification modeling approach given in Chessa *et al.* [3]. As is typical with X-FEM a level set is used to implicitly define the location of the phase interface. This facilitates the construction of the discontinuous enrichment function as well as defining phase constitutive properties.

In addition to the presentation of the enriched formulation of the problem a discussion of the construction of level set extensional velocities will be given. Accurate construction of the extensional velocity field is dependent on the projection of the velocity field on the zero level set to the neighboring nodes. An augmented Lagrangian method in conjunction with a MLS mesh free approximation is adopted that allows for a simple an natural method of performing this projection.

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Higher-Order Divergence-Free Discontinuous Galerkin Approximations to the Navier-Stokes Equations

Bernardo Cockburn
School of Mathematics, University of Minnesota, Minneapolis, MN, USA
cockburn@math.umn.edu

Guido Kanschat
Department of Mathematics, Texas A&M University, College Station, TX, USA
kanschat@tamu.edu

Dominik Schötzau
Department of Mathematics, University of British Columbia, Vancouver, Canada
schoetzau@math.ubc.ca

Abstract

We present a class of discontinuous Galerkin methods for the incompressible Navier-Stokes equations yielding exactly divergence-free solutions. Exact incompressibility is achieved by using divergence-conforming velocity spaces for the approximation of the velocities. No solenoidal basis functions are used, such that the method works as well in three dimensions and on not simply connected domains. The resulting methods are locally conservative, energy-stable, and optimally convergent. We present a set of numerical tests that confirm these properties.

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On *hp*-FEM Based on Generalized Eigenfunctions

Lenka Dubcova, Pavel Solin
The University of Texas at El Paso, 500 West University, El Paso, TX 79912
solin@utep.edu

Ivo Dolezel
Institute of Thermomechanics, Academy of Sciences of the Czech Republic,
Dolejskova 5, Praha, CZ-182 00, Czech Republic2
dolezel@fel.cvut.cz

Abstract

We introduce a new class of higher-order shape functions for the *hp*-FEM for second-order elliptic problems which are based on the generalized eigenfunctions of the Laplace operator [3]. Such elements were first discussed in one spatial dimension in [1]. Higher-order shape functions of this type possess remarkable simultaneous orthogonality both in the H_0^1 and L^2 products. This, in turn, has a positive influence on the sparsity and condition number of the corresponding mass and stiffness matrices.

In this study, we present a new set of higher-order shape functions for edge elements of electromagnetics which are based on the generalized eigenfunctions of the curl-curl operator. These functions are simultaneously orthogonal both in the curl-curl and L^2 products which makes them an excellent choice for the *hp*-FEM. We present numerical comparisons to other widely used families of higher-order shape functions for electromagnetics.

Finally, we show that the generalized eigenfunctions of the Laplace operator together with the generalized eigenfunctions of the curl-curl operator naturally fit into the De Rham diagram.

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The Verification of Coupled Heat and Moisture Transfer FEM Model

Dalibor Frydrych

Department of Electromechanical Systems, Faculty of Mechatronics and Interdisciplinary Engineering Studies Techical University of Liberec, CZECH REPUBLIC

dalibor.frydrych@tul.cz

Abstract

A buffer of compacted bentonite is planned to be used to prevent the movement of groundwater and the consequential escape of material from a geological repository for spent danger waste. Fluid flow, phase changes, chemical reactions, mechanical behavior of the buffer, rock and the waste canisters, and the heat produced by the waste constitute a coupled Thermo-Hydro-Mechanical-Chemical (THMC) system. The above-described processes are complex and cannot be currently solved in their full scope. Several teams from around the world are tackling this topic. Each team has developed their own approach to simplify the problem (an example of such simplification is limitation to 1D task).

Our team has chosen an approach based on a three-dimensional task coupled with heat and moisture transfer with sorption. The main part of this article deals with preparation of model. Appropriate physical equations describing heat and mass transfer through a porous material are chosen in the first step. Then, a mathematical model with boundaries and initial conditions is built. To use the finite element method, the weak formulation had to be derived, in order to use the finite element method. For spatial discretization, we use tetrahedrons with linear base functions. We use the implicit scheme for approximation of time derivatives, which provides sufficient numerical stability. Special attention is paid to solving of large, nonsymmetrical, structured and sparse set of linear equations in the implementation of the numerical model.

The results of experiment *GT40* from *Bechmark THM 1.2* described in [2] were used to verify our model. The results of the model are very close to the results measured during the experiment. Despite the simplifications the simulations and the observations are fairly consistent. The model succeeds in its aim to describe the essential features of the TH behavior i.e. heat transfer, moisture diffusion and adsorption/desorption.

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Structural Assessment Under Uncertain Parameters Via Interval Analysis

Oscar García
MiceLab - Modal Intervals & Control Engineering Laboratory, University of Girona, Department of Electronics, Informatic and Automation, Av. Lluís Santaló, s/n, 17071 Girona, Spain.
odgarcia@eia.udg.es

Josep Vehí
MiceLab - Modal Intervals & Control Engineering Laboratory, University of Girona, Department of Electronics, Informatic and Automation, Av. Lluís Santaló, s/n, 17071 Girona, Spain.
vehi@eia.udg.es

Abstract

An efficient health monitoring system for damage detection in civil engineering structures using on-line monitoring data is being developed to identify any possible damage in short time. The present work is based in the treatment of uncertainties, that is one of the basic common difficulties to face when modelling structures. Civil engineering structures contain uncertainty in their physical or geometric parameters, such as loading, Young's modulus, Poisson's ratio, length, etc. In this work, uncertainties are described in the way of intervals. The input parameters in civil engineering systems are replaced with intervals. The result of this process is an interval value, this means that parameters can take any value between the lower and the upper limit of the interval instead of a fixed real number.

A methodology, based on Interval Analysis ($\mathcal{IA}$) theory [1] applied to a numerical Constraint Satisfaction Problem ($\mathcal{CSP}$) [2], is implemented in the damage detection [2] and modelling system of a long term monitoring project in order to reach such objective. An algorithm is being developed to use such methodology with the obtained data. Such methodology has been first checked in laboratory with a simple reinforced concrete structure (loaded up to failure). The obtained results are useful to identify the cracking load. The majority of structures present a linear elastic behaviour during almost all life. However they tend to deteriorate, such degradation reflects on results obtained from the long term monitoring system. Structural assessment was performed in this case with success, enabling its application into real structures.

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A Continuous Galerkin Spectral Element Method for Triangular Elements

Francis X. Giraldo
Naval Research Laboratory, Monterey, CA
giraldo@nrlmry.navy.mil

Mark Taylor
Sandia National Labs., Albuquerque, NM
mataylo@sandia.gov

Beth Wingate
Los Alamos National Lab., Los Alamos, NM
wingate@lanl.gov

Abstract

This talk will focus on a generalization of the diagonal-mass-matrix Continuous Galerkin (DMM-CG) form of the spectral element method to triangular elements. DMM-CG is the "classical" spectral element method which has proven ideal for wave propagation and wave propagation dominated flows such as the Earth's atmosphere. For these problems, the diagonal mass matrix allows a fully explicit, arbitrarily high order finite element method. The method retains accuracy comparable to global spectral models while achieving unmatched parallel performance, scaling to tens of thousands of processors.

The DMM-CG method relies on the existence of highly accurate, specialized polynomial quadrature rules. These rules are only known to exist in the interval and its tensor products. Thus the method cannot be directly used with triangular or tetrahedral based elements. In this work, we present results from a generalization of DMM-CG for triangles. It is identical to the DMM-CG approach used in quadrilaterals, except for the use of a modified polynomial truncation. With this new truncation, suitable quadrature formulas can be found (although they must be found with numerical optimization). It is based on [1], where they presented results up to 5'th order accurate. We have used a new optimization algorithm [2] to find quadrature formulas which allow us to extended the method to even higher order. We will present results for the shallow water equations on the sphere showing that with these new quadrature formulas, DMM-CG in triangles can achieve the same convergence rates and levels of accuracy as DMM-CG in quadrilaterals.

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A New High-Order Accurate Method for Transient Dynamics Problems

Alexander Idesman
Department of Mechanical Engineering, Texas Tech University, Box 41021, Lubbock, TX
alexander.idesman@coe.ttu.edu

Abstract

A new high-order accurate time continuous Galerkin (TCG) method for elastodynamics is suggested [1]. A new two-field weak formulation (including unknown displacements and velocities) as well as space-time finite element approximations are used for the derivation of the final system of algebraic equations. For the structured meshes, the final system of algebraic equations can be also derived using the finite element approximations in space and polynomial approximations in time. The accuracy of the new implicit TCG method on structured meshes is increased by a factor of two in comparison to the standard TCG method and is one order higher than the accuracy of the standard time discontinuous Galerkin (TDG) method at the same number of degrees of freedom. The new method is unconditionally stable and has controllable numerical dissipation at high frequencies. An iterative multi-pass solver that includes the factorization of the effective mass matrix of the same dimension as that for the second-order methods is developed for the new TCG method. This iterative multi-pass solver requires only a few iterations in order to reach the accuracy of direct solvers. For example, two and three iterations are needed for the fourth and sixth order of accuracy, respectively. Simple 1-D numerical tests show that even with a direct solver, the new method reduces the computation time by 5-25 times in comparison to the existing second-order methods and is much faster than the TDG method. The new technique should be even more effective for multi-dimensional problems. It is also interesting to note the numerical results related to the problem of impact of an elastic bar against a rigid wall. It is known that the application of the traditional semi-discrete methods to this problem leads to oscillations in velocities and stresses due to the spurious high frequency response. The standard TDG method also yields spurious oscillations for this problem, especially on non-uniform meshes in space. However, numerical results show that the spurious oscillations can be excluded with the new TCG method. A new solution strategy combining numerical methods with zero (or small) and large numerical dissipation is developed for elastodynamics [1]. This strategy essentially increases the accuracy of numerical results for elastodynamics problems. An example of the application of the new strategy for wave propagation problems shows its effectiveness.

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Error Analysis in Finite Element Elastodynamic Problems using Function Space Approach

P. Jafarali , Somenath Mukherjee
Scientist, Aeronautical Development Agency, Bangalore 560017, India
jafaralnaf@rediffmail.com, somu@css.cmmacs.ernet.in
Mohammed Ameen
Assistant Professor, National Institute of Technology, Calicut 673601, India
ameen@nitc.ac.in
Gangan Prathap
Scientist-In-Charge, C-MMACS, Bangalore 560037, India
gp@cmmacs.ernet.in

Abstract
In the present work, the variational basis [1] for finite element analysis of elastodynamic problems has been examined using the principle of virtual work. It has been shown that derivation of a complete and accurate mathematical description of the nature of errors in free vibration analysis involves a simultaneous consideration of errors in both displacement and strains. Two fundamental important theorems on errors in variationally correct formulations in computational elastodynamics [1] have been discussed and illustrated with simple one-dimensional elements. A geometric interpretation of the behavior of these errors in approximate solutions from a variationally correct formulation has been presented using the Frequency-Error Hyperboloid. Furthermore, it has been shown using the Frequency-Error Hyperboloid that the variationally correct formulations with consistent mass matrices yield eigenfrequencies that are always higher than the analytical values, independent of domain discretisation. This is not necessarily true for variationally incorrect lumped mass formulations, and the computed eigenfrequencies can be higher or lower than or equal to the exact ones, depending on the discretisation scheme. It has been also shown that how the various elastodynamic variational statements, valid for the variationally correct consistent mass formulations, are violated when the lumped mass formulations are adopted. To see the consequences of employing lumped mass matrices a numerical test, called "the sweep test" is proposed in this paper. The boundedness of the approximate solution to the corresponding exact solution is a very important issue in error analysis. In the present work, it has been shown that, in the finite element elastodynamics approximation, any variational crimes, will lead to the violation of the projection theorem and the energy error rule for elastodynamics and also will lack the boundedness property of the finite element eigenvalues with the corresponding exact eigenvalues. An error indicator for the approximate finite element eigenvalues is also proposed in this paper.

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Program Generation for Polynomial Transforms in Unstructured Finite Element Computation

Robert C. Kirby
Department of Computer Science
The University of Chicago
1100 E. 58th St., Chicago, IL 60615
kirby@uchicago.edu

Markus Püschel
Department of Electrical and Computer Engineering
Carnegie Mellon University
5000 Forbes Ave., Pittsburgh, PA 15213
pueschel@ece.cmu.edu

Abstract

Efficient algorithms for applying operators lie at the heart of spectral element methods. Though algorithms are known [1], spectral methods are much more complicated on unstructured elements than on structured meshes. The critical issue is finding a combination of an expansion basis and a quadrature rule for which a function can be efficiently evaluated and differentiated at the quadrature points. Rather than looking for an explicit tensor product structure as in [1] or using dense matrix operations as in [4], we view the evaluation as a *polynomial transform*. In [2] it was shown that fast Fourier transform like algorithms for a large class of such transforms can be generated automatically using algebraic techniques from representation theory using a tool called AREP. The output of AREP can then be fed into SPIRAL [3] to obtain an actual optimized C implementation. We study the application of AREP and SPIRAL to automatically generate optimized black-box code for evaluating standard finite element bases. Experiments show that, for example, for evaluating derivatives of the third degree Lagrange basis functions at suitably chosen quadrature points, AREP and SPIRAL can reduce the operations count and the runtime by a factor of 2–4. These results suggest that it is possible to obtain reliable, elegant, high-performance finite element algorithms with a minimum of human involvement.

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Pole Condition: A Numerical Method for Helmholtz-Type Scattering Problems with Inhomogeneous Exterior Domain

Roland Klose, Frank Schmidt, Achim Schädle, Lin Zschiedrich

Konrad-Zuse-Institute Berlin Takustr. 7
D-14195 Berlin-Dahlem Germany
klose@zib.de

Abstract

A class of electromagnetic scattering problems is modeled by the Helmholtz equations on unbounded domains. A central task in the numerical solution of such problems is the implementation of transparent boundary conditions. In this talk we present a numerical realization of the pole-condition method [2], a new approach to transparent boundary conditions. The pole-condition method is designed as a transparent boundary condition which also handles certain types of inhomogeneous exterior domains and provides a representation formula for the field in the exterior domain. Furthermore, a flexible adaption to many polygonal geometries for the interior domain is possible [1]. We present an algorithmic realization of the pole-condition method in a finite element setting. An extension to the Maxwell equations is discussed. Numerical examples illustrate the convergence of the method.

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Towards Combining Interval and Probabilistic Uncertainty in Finite Element Methods

Vladik Kreinovich[1], Roberto Araiza[2], Matthew G. Averill[2]
Departments of [1]Computer Science and [2]Geological Sciences
University of Texas, El Paso, TX 79968, USA
vladik@utep.edu, raraiza@utep.edu, averill@geo.utep.edu

Abstract

Most traditional FEM techniques are based on the assumption that we know the *exact* equations and the *exact* values of the parameters of these equations. In practice, we only know the *approximate* values of the corresponding parameters. To make the FEM results practically useful, we must be able to estimate how the uncertainty in the parameters of the system can affect the result y of applying the FEM techniques. Some such estimations assume that we know the exact *probability* distributions corresponding to all the imprecisely known parameters; see, e.g., [1]. In such *stochastic FEM* situations, we can compute the probabilities of different values of the difference $\Delta y = y - y_0$ between the actual values y and the results y_0 of the traditional FEM.

In many practical situations, we do not know these probabilities. For example, in civil engineering, we often only know the lower and upper bounds on the Young module, but the probabilities of different values within the corresponding *interval* may depend on the manufacturing process and may be thus drastically different from one building to another. If we want, e.g., to guarantee the building's reliability, we must find the *interval* $[-\Delta, \Delta]$ of possible values of Δy; see, e.g., [2] for the corresponding *interval* FEM techniques. In engineering and geosciences, we sometimes have *both* interval and probabilistic uncertainty: e.g., for manufacturing-related parameters, we may only know intervals of possible values, but for weather-related parameters, we also know the probabilities of different values (e.g., from the weather records). In such situations, we want to know the bounds $\Delta(p)$ that contain Δy with different probabilities p.

In this talk, we describe algorithms for producing such bounds. For example, we can use Monte-Carlo techniques to simulate parameters with known probability distributions. For each such simulation, we use interval FEM techniques to get the interval bounds for the resulting FEM inaccuracy. After several simulations, we get the resulting bounds distribution, from which we can find, for each probability p, the desired bound $\Delta(p)$.

As a case study, we use a seismic inverse problem from the geosciences.

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Arbitrary-Level Hanging Nodes in the *hp*-FEM

Pavel Kus, Pavel Solin, Jakub Cerveny
The University of Texas at El Paso, 500 West University, El Paso, TX 79912
pkus@utep.edu, solin@utep.edu, jcerveny@utep.edu

Abstract

It was shown in [1] that the employment of arbitrary-level hanging nodes allows for a great simplification of automatic adaptive strategies in the *hp*-FEM. The reason is that the choice of optimal *hp*-refinement can be done fully locally in elements. In other words, when refining an element, adjacent elements never have to be refined. Such additional refinements which are needed to satisfy mesh regularity rules are called *forced refinements*, and they can slow down the convergence rates significantly.

This study is concerned with an extension of 2D results [1] to hexahedral elements. Compared to the two-dimensional case, the 3D setting is much more complicated and a number of interesting new problems appear. These problems are presented, their solution described, and numerical examples are shown.

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On the Design of High-Resolution Finite Element Schemes Satisfying the Discrete Maximum Principle

Dmitri Kuzmin
Institute of Applied Mathematics (LS III), University of Dortmund
Vogelpothsweg 87, D-44227, Dortmund, Germany
kuzmin@math.uni-dortmund.de

Abstract

An algebraic approach to the design of high-resolution schemes for convection-dominated flow problems [1] is revisited and extended to quadratic finite elements. It is explained how to get rid of nonphysical oscillations and to remove excessive artificial diffusion in regions where the solution is sufficiently smooth. To this end, the discrete operators resulting from a standard Galerkin discretization of the troublesome convective terms are modified so as to enforce the discrete maximum principle without violating mass conservation. In the case of quadratic elements, the diffusive/antidiffusive flux between two nodes is redefined to include the function values at intermediate nodes (if any) so as to minimize the amount of artificial diffusion induced by elimination of negative off-diagonal coefficients. The differences between the treatment of linear and quadratic FEM approximations are highlighted.

A family of algebraic flux correction schemes is derived on the basis of a node-oriented limiting strategy which traces its origins to the multidimensional flux-corrected transport (FCT) algorithm [4]. A general-purpose flux limiter is designed to provide an accurate treatment of stationary and time-dependent problems alike [2]. Last but not least, the P_1-version is combined with adaptive mesh refinement techniques, whereby flux/slope limiters serve as error indicators [3]. Numerical results are presented for scalar conservation laws as well as for the Euler and Navier-Stokes equations in two and three dimensions.

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Toward a Hybrid-Trefftz Finite Element with a Hole for Elastoplasticity

N. Leconte, B. Langrand
Department of Solid and Damage Mechanics, ONERA-Lille, 5 boulevard Paul Painlev, F-59045 Lille cedex, France
nicolas.leconte@onera.fr, bertrand.langrand@onera.fr

E. Markiewicz
University of Valenciennes, Industrial and Human Automatic Control and Mechanical Engineering Laboratory (UMR CNRS 8530), Le Mont-Houy, 59300 Valenciennes, France
eric.markiewicz@univ-valenciennes.fr

Abstract

The paper deals with the modelling of riveted assemblies for full-scale complete aircraft crashworthiness. Recent comparisons between experiments and FE computations of bird impact onto aluminium riveted panels have shown that macroscopic plastic strains were not sufficiently developed (and localised) in the riveted shell FE. Consequently, FE models never suceeded in initialising and propagating the rupture in the sheet metal plates and along rivet rows as shown in the experiments, without tuning the input data (especially the damage and failure properties of the riveted shell FE). To compute the behaviour of the assembly correctly, a model dealing with geometrical defects effects is firstly introduced. It appeared necessary to investigate on FE techniques such as Hybrid-Trefftz Finite Element Method (H-T FEM). Indeed, perforated FE plates developed for elastic problems, based on a Hybrid-Trefftz formulation, have been found in the open literature. Our purpose is to find a way to extend this formulation so that the super-element can be used for crashworthiness. To reach this aim, the main features of an elastic Hybrid-Trefftz plate are presented and are then followed by a discussion on the perspectives of extension.

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Variability Response Function for Stochastic Transient Heat Conduction with Random Conductivity

Mladen Meštrović
University of Zagreb, Faculty of Civil Engineering, Kačićeva 26, 10000 Zagreb, Croatia
mestar@grad.hr

Abstract

The concept of variability response function [1], [2] based on the weighted integral method [3] is extended to one-dimensional transient heat conduction. The thermal conductivity of the structure is considered to be two-dimensional (in one-dimensional space-time domain) [4], homogenous, stochastic field. The stochastic element conductivity matrix is decomposed into deterministic part and stochastic part of element conductivity matrix. The stochastic part of element conductivity matrix is expressed as linear function of the random variables (weighted integrals) with zero-mean property.

The concept of the variability response function is used to compute the upper bound of the response variability (response temperature). The first and second moment of stochastic thermal conductivity are used as input quantities for the description of the random property. The response temperature is calculated using first-order Taylor expansion approximation of the variability response function. The variability of the response temperature is represented in the terms of the second moment of the response temperature and related coefficient of variation.

The k-time-step-randomness method is introduced to reduce the computational effort. The algorithm for calculation with variability of the response temperature from just last k time steps is given. Numerical examples are provided for different values of k. Calculated results with less computational effort are compared with results calculated with complete random history.

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FEM for Schrödinger Equation with Rashba Effect

Yuu Miyagawa
Graduate School of Science and Technology, Chiba University
1-33 Yayoi-cho, Inage-ku, Chiba, 263-8522, Japan
myuu@graduate.chiba-u.jp

Tsuyoshi Ueta
Graduate School of Science and Technology, Chiba University
1-33 Yayoi-cho, Inage-ku, Chiba, 263-8522, Japan
ueta@faculty.chiba-u.jp

Abstract

Recently the spin electronics using both the electronic charge and the spin of electrons in a solid show rapid development. Mesoscopic systems have a size of about 1nm to 100nm and characterized by coherent property. One of mesoscopic systems, ballistic systems are shorter than the mean free path in which an electron move not being scattered. When a control of spin current is achieved in mesoscopic systems, various quantum effect phenomena and appearances of new device using spin are expected [1]. However up spin and down spin electrons make the same contribution to conductance without effects depended on spin. There are a magnetic field, magnetic scatterers and rashba effect and so on in such effects.

Ballistic systems which magnetic scatterers is introduced is analyzed by the boundary element method (BEM) in 2-dimensional electron systems (2DES). The BEM is extended to treat systems including many pointlike magnetic scatterers in which a volume integral term is evaluated using an approximation [2]. As a result the high spin polarization is obtained the model arranged in the shape of a lattice. Other effect, the rashba effect appear in a 2DES applied a perpendicular electric field. The rashba spin-orbit interaction is

$$H_{RSO} = \frac{\alpha}{\hbar}(\boldsymbol{\sigma} \times \boldsymbol{p}) \cdot \boldsymbol{v}, \tag{1}$$

where α is the intrinsic coefficient, $\boldsymbol{\sigma}$ is the Pauli matrices, $\boldsymbol{p}$ is a momentum and $\boldsymbol{v}$ is a unit vector perpendicular to a 2DES. Volume integration appears according to Eq.(1), it is hard to apply a BEM. We adopt and extend a FEM in order to evaluate Eq.(1).

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Finite-Element Dynamic Analysis of Transportation Systems with Interval Uncertainty

Mehdi Modares
Department of Civil Engineering
University of New Hampshire, Durham, NH 03824
mxm206@case.edu

Robert L. Mullen
Department of Civil Engineering
Case Western Reserve University, Cleveland, OH 44106
rlm@case.edu

Abstract

In transportation engineering, dynamic analysis is an essential procedure for designing a system. Moreover, the performance of the designed system must be guaranteed over its lifetime. However, in current procedures of dynamic analysis for transportation systems, the presence of uncertainty in characteristics of the structure and applied forces is not explicitly considered. In this work, a new method is developed for the finite-element based dynamic analysis of uncertain structure subjected to uncertain loads induced by passage of moving vehicles. First, an interval formulation is used to quantify the uncertainty present in the structures mechanical characteristics and/or magnitude of dynamic force. Then, having the interval parameters, the upper-bounds on the structures response is obtained using Interval Response Spectrum Analysis (IRSA) by which the results may be used for design purposes. An example problem that illustrates the behavior of the method and a comparison with Monte-Carlo simulations and combinatorial solution are presented.

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Solving Regularly and Singularly Perturbed Reaction-Diffusion Equations in Three Space Dimensions

Peter K. Moore
Southern Methodist University

Abstract

I will present a fixed, high-order h-refinement finite element algorithm for solving singularly- and regularly-perturbed reaction-diffusion equations in three space dimensions. Spatial adaptivity is coupled with continuation and the nonlinear solver *NITSOL*. Good initial guesses for the nonlinear solver are obtained using continuation in the small parameter ϵ. Two strategies allow adaptive selection of ϵ. The first depends on the rate of convergence of the nonlinear solver and the second implements backtracking in ϵ. Finally a simple method is used to select the initial ϵ. Several examples illustrate the effectiveness of the algorithm.

Constructive Error Estimates in the Finite Element Methods with Applications to Verification of Solutions for Nonlinear PDEs

Mitsuhiro T. Nakao
Faculty of Mathematics, Kyushu University 33
Fukuoka 812-8581, JAPAN
mtnakao@math.kyushu-u.ac.jp

Yoshitaka Watanabe
Computing and Communications Center, Kyushu University
Fukuoka 812-8581, JAPAN
watanabe@cc.kyushu-u.ac.jp

Abstract

We have been devoted for almost two decades to studying the numerical verifications of solutions to elliptic partial differential equations(PDEs). Our approach is based on the combination of fixed point theorems in functional spaces and the constructive error estimations of the finite element methods(FEM) and the spectral methods. While the FEM is one of the most convenient and strong numerical methods for PDEs and many error analyses have been derived from the theoretical point of view, there are very few works on the computable error estimates in, mathematically rigorous, a posteriori sense.

In this talk, we first consider the constructive a proiri error estimates in FEM for Poisson's equation and for bi-harmonic problems. Next, as an application of the results, we show a numerical verification method of solutions for nonlinear elliptic problems and Navier-Stokes equations as well as other applications. The special emphasis of our method is that we can obtain the finite element solution with guaranteed error bounds even if we have no information about the existence of exact solutions for the original equations such as noncoercive or nonlinear problems.

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On Solvability of a Hydro-Mechanical Problem Based on Contact Problem with Visco-Plastic Friction in Bingham Rheology

Jiri Nedoma, Lubos Tomasek
Institute of Computer Science AS CR,
Prague, Czech Republic

Abstract

The contribution will deal with the solvability of contact problem with visco-plastic friction in the visco-plastic Bingham rheology. The generalized case of bodies of arbitrary shapes being in mutual contact will be assumed. To prove the existence of the solution the multipliers will be introduced and the regularization technique will be used. Numerical solution of the problem will be based on the semi-implicit scheme in time and finite element approximation in space. It can be shown that the scheme is convergent and stable. Two model problems will be presented, i.e. (i) the model problem of loaded long bone, (ii) the model problem of nonstable slope.

ad (i) Abnormality in human joint biomechanics is the main cause of the degenerative disease development, therefore the description and correction of the joint biomechanics are essential for adequate treatment. Understanding of the transmission of loads through the long bone starts to be substantial for biomechanical analyses of human joints and their artificial replacements. The stress/strain analyses of long bones based on the Bingham rheology facilitate analyses of transmission of loads also through marrow and soft tissues.

ad (ii) The idea of the proposal how to predict the future situation in the endangered regions is based on the mathematical simulation of coupled hydro-mechanical and hydro-dynamical processes together with the climatic processes in the endangered region during the strong hurricane and the consequences of the enormous quantity of water in this region onto the stability of slopes in this region and onto the security of this region.

The mathematical model discussed is based on the visco-plastic Bingham rheology. The Bingham rheology is characterized by two parameters - threshold of plasticity g and threshold of viscosity μ. Since the threshold of plasticity g is from $[0, +\infty]$, the environment can be described from liquid ($g = 0$), through the liquid saturated more or less by minerals and rocks upto the absolutely rigid rocks ($g \to +\infty$). Therefore, we can model all types of rocks, which are loaded by external forces as loading forces induced by the hurricane, weights of rainfalls, weights of materials in the rockcovers, weights of industrial buildings, etc.

A Hybrid Finite Element Method for Stress Concentration in Composites under Dynamic Loads

R. Paskaramoorthy, R. G. Reid
School of Mechanical, Industrial and Aeronautical Engineering
University of the Witwatersrand
South Africa
Ratnam.Paskaramoorthy@wits.ac.za, Robert.Reid@wits.ac.za

Abstract

Fibre-reinforced composites are being increasingly used in a number of applications in aerospace, automobile and other industries. In analysing the strength of composite materials, an important consideration is the magnitude of stress concentration in the matrix at the fibre/matrix interface. Debonds generally initiate at this interface, which need to be studied thoroughly. An important factor that contributes to debonding is stress concentration. While numerous studies have considered the stress concentration at the fibre/matrix interface under static loads [1], [2], studies that considered the effect of dynamic loads are very scarce. In this study, a hybrid technique is developed in order to study the steady-state stresses resulting from a dynamic loading in a single fibre. The problem is considered as consisting of two interacting systems, a bounded interior region containing the fibre, and an unbounded exterior region. The geometry of the fibre is assumed to be spheroidal. The interior region is modelled by using conventional isoparametric finite elements, and the exterior region is modelled by using spherical eigen functions. Numerical results reveal that dynamic stresses could be up to 100% greater than the static values. The results also indicate that the region of maximum von Mises equivalent stress within the matrix varied along the fibre-matrix interface, but also occurred at interior points away from the interface for some frequencies. The latter observation is significant since cracks in a ductile matrix could originate at regions other than the fibre-matrix interface.

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Numerical Solution of FEM Equations with Uncertain Functional Parameters

Andrzej Pownuk
Department of Mathematical Sciences The University of Texas at El Paso 500 West University Avenue El Paso, Texas 79968-0514, USA
andrzej@pownuk.com

Abstract

Many numerical methods like FEM, BEM, FVM, FDM etc. leads to the problem of solution parameter dependent system of equation in the following form [1, 2]

$$A(h)u = B(h) \tag{1}$$

where A is some matrix (e.g. a stiffness matrix), B is some vector (e.g. a load vector), u is a vector of unknown (e.g. unknown nodal values) and $h = [h_1, ..., h_m]^T$ is a vector of some parameter (e.g. Yang modulus, length, volume, force etc.). In engineering practice very often we do not know the exact values of the parameters h_i. In order to create mathematical model of such systems several different method of modelling of uncertainty can be applied (e.g. probabilistic methods, imprecise probability methods, fuzzy methods, interval methods etc. [1]).

In some cases parameters h_i are not only numbers but also functions. In that case in the equation (1) we have uncertain functional parameters. One of the simplest example of such parameter is a distributed load. The solution of the equation (1) can be defined in the following way

$$u(\tilde{h}) = \{u : K(h)u = Q(h), h \in \tilde{h}\} \tag{2}$$

where $\tilde{h}$ is some functional space.

In this paper some new method of finding the solution (2) is presented. The method is based on sensitivity analysis method [1] and the concept of functional derivative [3]. Several numerical examples will be also presented.

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Discontinuous Galerkin Deforming Grid(DGDG) Method for Large Deformation Viscoelastodynamics

Saurabh Srivastava, Abani Patra
605 Furnas Hall, Department of Mechanical and Aerospace Engineering, University at Buffalo, Buffalo, NY-14206, U.S.A
{ss228,abani}@eng.buffalo.edu

Abstract

In this work, we apply an explicit Discontinuous Galerkin(DG) method for long duration simulation of large deformation problems in elastodynamics and nonlinear viscolelasticity. We will illustrate that our method works very well for the simulation of the latter class of problems in conjunction with an updated Lagrangian scheme to track material deformation. The local conservation property of DG methods also aids long duration simulations. The mixed methodology in the context have independent interpolations for displacement and strain(or stress) variables. An appropriate choice of interelement flux, also allows the condenstation of strain DOF's at element level. However, a compatible approximating spaces for primary variable('u') and strain(or stress) is rather difficult to design, since it involves the interpolation of symmetric gradient(stress) as noted by Arnold and Winther in [1]. Thus to alleviate this issue we interpolate full displacement gradient field which has no contraints of symmetry, and extract the symmetric part as desired. Further simplification to the construction of compatible approximation is achieved by using a variational formulation that contains only gradient operators instead of a divergence operator as shown in [2]. With these simplifications at hand we discretize the domain using Quads and use higher order hierarchical polynomials for interpolating both primary variable('u') and its gradient(D). Thus we attempt to solve the elastodynamics problem in the following weak form, Find $(u_i,D_{ij}) \in H^1(\Omega_e) \times L_2(\Omega_e)$ such that $\forall(\omega_i, S_{ij}) \in H^1(\Omega_e) \times L_2(\Omega_e)$ $\int_{\Omega_e^t} D_{ij} : S_{ij} d\Omega - \int_{\Omega_e^t} u_{i,j}.S_{ij} d\Omega = 0$ $\int_{\Omega_e^t} \rho.\dot{v}_i.\omega_i d\Omega + \int_{\Omega_e^t} \sigma(D_{ij}) : \omega_{i,j} d\Omega = \oint_{\partial\Omega_e^t} \{\sigma(\hat{D}_{ij})\}.n_j.\omega_i ds + \int_{\Omega_e^t} f_i.\omega_i d\Omega$. For stability, the well known $Inf - Sup$ condition must be satisfied by the resulting operator([3]). We will discuss a nonlinearly stable time stepping scheme in updated Lagrangian framework and display convergence results of our method for linear elastodynamics in 2D. This will be followed by extension of the framework to incorporate complex nonlinear Viscoelastic constitutive models. We also demonstrate a stable and convergent mixed method for elastostatics which is implemented with great ease.

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Multiscale Discontinuous Galerkin Methods for Modeling Flow and Transport in Porous Media

Shuyu Sun
Department of Mathematical Sciences, Clemson University
O-221 Martin Hall, Clemson, SC 29634-0975, USA
shuyu@clemson.edu

Abstract

Subsurface flow and transport phenomena may involve multiple time and spatial scales, long simulation time periods, and many coupled nonlinear components. In particular, the advection-dominated component and the coupling of transport with nonlinear reactions often result in sharp concentration fronts. The latter requires steep gradients to be preserved with minimal oscillation and numerical diffusion. Thus, multiscale treatment and dynamic adaptivities are essential for accuracy and efficiency.

Discontinuous Galerkin (DG) methods are specialized finite element methods that utilize discontinuous spaces to approximate solutions. Derived from variational principles by integration over local cells, the methods are locally mass conservative by construction. Boundary conditions and interelement continuity are weakly enforced in DG; consequently, DG methods have small numerical diffusion and little oscillation. In addition, they handle rough coefficient problems and capture the discontinuity in the solution very well by the nature of discontinuous function spaces. Moreover, the treatment of full-tensor permeability (for flow) and diffusion-dispersion (for transport) tensors is flexible and efficient in primal DG methods. In this presentation, we will consider the primal DG methods with adaptive and multiscale implementations as applied to flow and transport in porous media.

Numerical Approximation of Non-Newtonian Fluid Flow by the Higher-Order Finite Element Method

P. Sváček [1]
Czech Technical University, Faculty of Mechanical Engeneering,
Karlovo nám. 13, Praha 2, 121 35
Petr.Svacek@fs.cvut.cz

Abstract

Various industrial slurries are concentrated mixtures of very small particles and grains in water. Generally, these suspensions are non-Newtonian fluids exhibiting a yield stress that needs to be overcome for the flow to take place, see [1]. Efforts to introduce large-scale numerical simulations to rheology began in the late 1970s. Recently it has become possible to simulate a broad variety of flows with many different constitutive equations. The use of the finite element method for non-Newtonian flows was studied by [2].

In this paper the main interest will be paid to the proper simulation of non-Newtonian fluid flows. We will discuss the choice of the mathematical model, the weak formulation, the approximation of the free-surface condition, possible sources of instabilities, choices of finite elements, stabilization procedures based on the streamline-upwind/Petrov-Galerkin (SUPG) method and the Galerkin-Least-Squares (GLS) method, for the application on Newtonian incompressible flow see, e.g., [3].

Furthermore, the problem of free-boundary will be discussed. A number of special techniques that can be applied for free surface prediction have been developed through years. Two main trends exists: interface tracking methods and interface capturing methods. Short overview of both types will be given. The main attention will be paid to the Arbitrary Lagrangian-Eulerian (ALE) method. The influence of the mesh deformation will be discussed and the application of ALE method on free-surface problem will be shown.

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A Stochastic Approach to Validation of Finite Element Models

Angel Urbina
Sandia National Laboratories; PO Box 5800; Mail Stop 0828; Albuquerque, NM 87185-0828
aurbina@sandia.gov

Thomas L. Paez
Sandia National Laboratories; PO Box 5800; Mail Stop 0557; Albuquerque, NM 87185-0557
tlpaez@sandia.gov

Abstract

We are continuously improving our capability to accurately model complex structures and with the help of sophisticated computer codes that operate on massively parallel platforms, we can obtain solutions to complex, nonlinear problems that were unattainable a few years ago. Deterministic finite element models which traditionally have been exercised at some assumed nominal values of the parameters of interest have been used reliably for many years. Our current analyses take into account the stochastic nature of excitations and the stochastic nature of structures themselves. As mathematical models increase in complexity we expect for them to simulate reality with increasing accuracy and in some sense this will need to include the randomness found in actual physical systems. Yet, we need to assess the level of accuracy provided by our models and analyses, relative to experimentally measured systems. This is complicated by the fact that real physical systems are stochastic, and in addition, measurements of their behavior are noisy. Therefore, development of methods for the validation of mathematical models needs to take into account both probabilistic and statistical variation in experimental systems and the mathematical models. At Sandia National Laboratories, we have formalized a process to validate finite element models and make quantitative assessments of how good a model is relative to an intended use. We start with a definition of validation which states that validation is the "process of determining the degree to which a computer model is an accurate representation of the real world from the perspective of the intended model applications." [1] [2]. This work summarizes an approach to model validation that takes into account the stochastic nature of physical systems and randomness built into their mathematical models. A numerical example involving a complex aerospace component is presented.

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Local Discretizaton Errors for Boundary Element Analysis

B.F. Zalewski
Department of Civil Engineering
Case Western Reserve University
Cleveland, OH 44106
bxz10@case.edu

R.L. Mullen
Department of Civil Engineering
Case Western Reserve University

Abstract

Finite element and finite difference methods are widely used to solve partial differential equations that govern engineering systems. Boundary element methods are an alternate approximation of partial differential equations. It considers fundamental solutions of partial differential equations to reduce the dimension of the approximation. In this work, a method is developed to account for the discretization and truncation error in the boundary element method. The errors are treated using the concept of interval arithmetic. Interval methods have been previously explored in finite element method to model the uncertainties in boundary conditions, material properties as well as the truncation error. The interval boundary element method developed provides worst case bounds for each point on the boundary. From the boundary condition bounds, worst case bounds for any point in the domain can be calculated. Examplars are presented and the effectiveness ratios of the computed bounds are presented.

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